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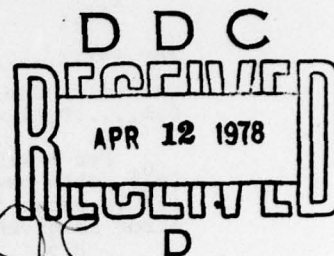
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ALIASING ERROR IN THE MULTIRATE IMPLEMENTATION OF NARROWBAND FILTERS[†]

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ABSTRACT

Narrowband filters are inherently of high order, so they require a high computation rate if implemented directly. Multirate filters are capable of effectively approximating narrowband characteristics. Such an implementation may require a much reduced rate of computation. This paper derives an expression for the system error of this approach. This error consists of two components, one due to the filter approximation and the other due to aliasing. The role of aliasing error in the design of multirate filters is then considered. Examples are presented.

I. INTRODUCTION

Multirate filters, those filters composed of cascaded decimators and interpolators, have been shown to be useful in implementing narrowband lowpass filters. Bellanger et al. in [1] used multirate filters composed of half band decimators and interpolators for lowpass filtering and they demonstrated that computational savings could be realized. Rabiner and Crochiere in [2] and [3] found multirate lowpass implementations which minimized either multiplication rate or coefficient storage. Both of these works assumed that the aliasing error was negligible.

In this paper, we calculate the system error of multirate implementation of lowpass filters. This error consists of two components, one due to filter approximation and the other due to aliasing. By using this result, design rules can be formulated which limit the aliasing error in these filters. Examples of multirate filters so designed are given.

[†]This research is sponsored by the Air Force Office of Scientific Research, Air Force Systems Command, USAF, under Grant No. AFOSR-76-3083.

II. THE SYSTEM ERROR FOR AN N-STAGE MULTIRATE FILTER

Let $\{x_n^0\}$ be the input to and $\{y_n^d\}$ the output of a filter with transfer function $F_d(z)$, as illustrated by the upper signal path of Fig. 1. An N-Stage multirate implementation of this filtering, shown as the lower signal path of Fig. 1, is composed of N generalized decimators followed by N generalized interpolators. Its output is $\{y_n\}$ which is to be close to $\{y_n^d\}$.

A generalized decimator consists of a filter $G_i H_i(z)$ followed by a decrease in the sampling rate by a factor of D_i . It reduces to a simple decimator if H_i is a lowpass filter. A generalized interpolator consists of an increase in the sampling rate by a factor of D_i followed by the filter $\beta_i G_i(z)$.

Let H_i and G_i be linear phase FIR filters of orders N_{Hi} and N_{Gi} respectively. The H_i are symmetric, and only one output of H_i is required for every D_i inputs. Only one of each D_i inputs to filter G_i is non zero. By taking advantage of these properties, the multirate filter requires

only $\sum_{i=1}^N (N_{Gi} + N_{Hi}/2) / \prod_{j=1}^i D_j$ multiplications per output. If this is less than the multiplication rate of a comparable direct implementation, a computational saving is realized by using the multirate structure.

In the lower path of Fig. 1, the sampling rate is increased and decreased according to the equations:

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$$x_{n_i}^{2i} = x_{D_i n_i}^{2i-1} + \theta_i \quad (1)$$

$$x_{n_{i-1}}^{4N-2i+1} = \begin{cases} x_{n_i}^{4N-2i} & , n_{i-1} = D_i n_i + \theta_i \\ 0 & , \text{otherwise} \end{cases} \quad (2)$$

for $i=1,2,\dots,N$ where θ_i is an integer valued random variable taking on the values $0,1,\dots,D_i-1$ with equal probability.

For a wide sense stationary input $\{x_n^0\}$, with power spectral density $\Phi[\exp(j\omega)]$, it is shown in [4] that the mean square error is given by

$$E\{(y_n - y_n^d)^2\} = \int_{-\pi}^{\pi} \left| \prod_{i=1}^N H_i[g_i(\omega)] G_i[g_i(\omega)] - F_d[g_1(\omega)] \right|^2 \times \Phi[g_1(\omega)] d\omega / 2\pi + \int_{-\pi}^{\pi} \left| \sum_{i=1}^N \prod_{j=1}^N G_j[g_j(\omega)] H_i[f_i(\omega)] \right|^2 \Phi[f_1(\omega)] d\omega / 2\pi \quad (3)$$

for $\prod_{i=1}^N \alpha_i \beta_i = \prod_{i=1}^N D_i = D$ where

$$g_i(\omega) = \exp(j\omega D_1 \dots D_{i-1}) \quad (4)$$

$$f_i(\omega) = \exp(j\omega \prod_{j=1}^{i-1} D_j + j2\pi \sum_{k=i}^N n_k / \prod_{j=1}^k D_j) \quad (5)$$

and the product $[D_1 \dots D_{i-1}]$ is defined to be 1 for $i=1$. The symbol \sum_N represents N summations; one for each n_i . The range of summation of n_i is from 0 to D_i-1 , but the term where all n_i equal zero is excluded from the summation.

The first term in (3) is the error due to approximation of $F_d[\exp(j\omega)]$ by $\prod_{i=1}^N H_i[g_i(\omega)] G_i[g_i(\omega)]$. We call this the filter approximation error and denote it by e_F . The second term in (3) is the error due to aliasing generated by the sampling rate changes. We call this the aliasing error and denote it by e_A .

For acceptable filter approximation, we require that the magnitude of

$\prod_{i=1}^N H_i[g_i(\omega)] G_i[g_i(\omega)]$ be within δ_p of $F_d[\exp(j\omega)]$ in its passband and within δ_s of $F_d[\exp(j\omega)]$ in its stopband. For acceptable aliasing error we require that

$$e_A \leq E\{(x_n^0)^2\}/S \quad (6)$$

for some specified S . Since the aliasing

is correlated to the signal it is more objectionable than noise which is not. Thus a reasonable S would make e_A comparable to, but somewhat less than the roundoff noise. By picking

$$\omega_{sH_i} = \omega_{sG_i} = 2\pi/D_i - D_1 \dots D_{i-1} \omega_s \quad , \text{ and} \quad (7)$$

$$\omega_{sHN} = \omega_{sGN} = D_1 \dots D_{N-1} \omega_s \quad (8)$$

an approximation for e_A can be made which

is valid for a reasonably smooth $\Phi[\exp(j\omega)]$. This approximation allows us to control the aliasing error by simple choices of the stopband ripples of $H_i[\exp(j\omega)]$ and $G_i[\exp(j\omega)]$. The procedure of Crochiere and Rabiner in [5] is used to pick the D_i subject to the restrictions of (7) and (8). The rest of the design parameters of the $H_i[\exp(j\omega)]$ and the $G_i[\exp(j\omega)]$ are then picked to satisfy the filter approximation requirement. The details can be found in [4].

III. EXAMPLES

We present here three examples of multirate filter design.

Example 1. The specifications are:

$$\omega_p = .05\pi, \delta_p = .01, \omega_s = .1\pi, \delta_s = .001, S = 10^5.$$

By using the design program of McClellan et al. [6], a filter of order of 110 was found to satisfy the specifications. The design parameters for a one stage multirate filter are:

$$\omega_{pH} = .05\pi, \delta_{pH} = .005, \omega_{sH} = .1\pi, \delta_{sH} = .0046;$$

$$\omega_{pG} = .05\pi, \delta_{pG} = .005, \omega_{sG} = .1\pi, \delta_{sG} = .0015; D=10.$$

Filters were designed to these specifications. The orders of H and G are 96 and 110 respectively. The optimization procedure of Crochiere and Rabiner in [5] selects as optimal $D_1=5, D_2=2$. For this two stage implementation, our filter design parameters are

$$\omega_{pH1} = .05\pi, \delta_{pH1} = .0025, \omega_{sH1} = .30\pi, \delta_{sH1} = .0046;$$

$$\omega_{pG1} = .05\pi, \delta_{pG1} = .0025, \omega_{sG1} = .30\pi, \delta_{sG1} = .0018;$$

$$\omega_{pH2} = .25\pi, \delta_{pH2} = .0025, \omega_{sH2} = .50\pi, \delta_{sH2} = .0046;$$

$$\omega_{pG2} = .25\pi, \delta_{pG2} = .0025, \omega_{sG2} = .50\pi, \delta_{sG2} = .0018.$$

Filters were designed to meet these parameters using [6]. The orders of H_1, G_1, H_2 and G_2 were 20, 22, 23, 25.

Figure 2 illustrates

$\prod_{i=1}^N H_i[g_i(\omega)] G_i[g_i(\omega)]$ for the one stage implementation. Figure 3 illustrates it for the optimal two stage implementation.

Table 1 gives a comparison of the multiplication rates of the various implementations.

Example 2. The specifications are $\omega_p = .0833\pi$, $\delta_p = .31$, $\omega_s = .1667\pi$, $\delta_s = .155$, $S=10^3$. The Crochiere and Rabiner procedure generates $D_1=3$, $D_2=2$ as the optimal choices of the D_i . Table 1 compares computation rates for the direct, one stage, and optimal implementations.

Example 3. The specifications are $\omega_p = .0833\pi$, $\delta_p = .31$, $\omega_s = .1667\pi$, $\delta_s = .155$, $S=10^5$. Table 1 compares the multiplication rates for the direct, one stage and two stage implementations.

IV. CONCLUSIONS

Aliasing error arises in a multirate filter as a result of sampling rate changes. We have calculated the aliasing error in a multirate filter, and designed multirate filters which impose a bound on aliasing error while meeting the filter approximation specifications. Examples 2 and 3 demonstrate that this multirate filter may not have a computational advantage over the direct implementation. In many cases, however, the multirate filter will have a computational advantage as it does in Example 1.

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Table 1. Comparison of Multiplication Rates of the Examples

Implementation	Example 1	Example 2	Example 3
Direct form	55.0	6.0	6.0
One-Stage	15.8	7.0	10.0
Two Stage (optimum)	10.1	6.0	9.8

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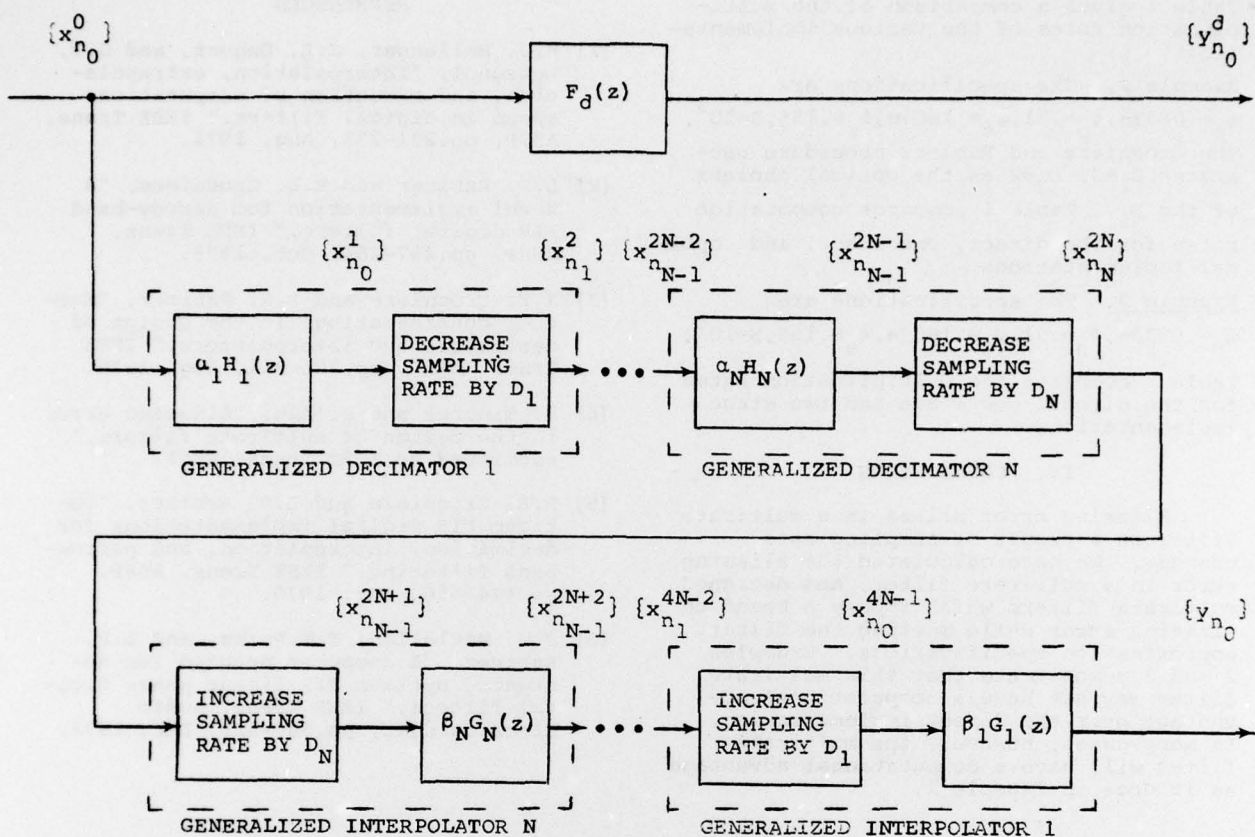


Fig. 1. N-STAGE MULTIRATE IMPLEMENTATION
(top path: desired filtering
bottom path: multirate implementation)

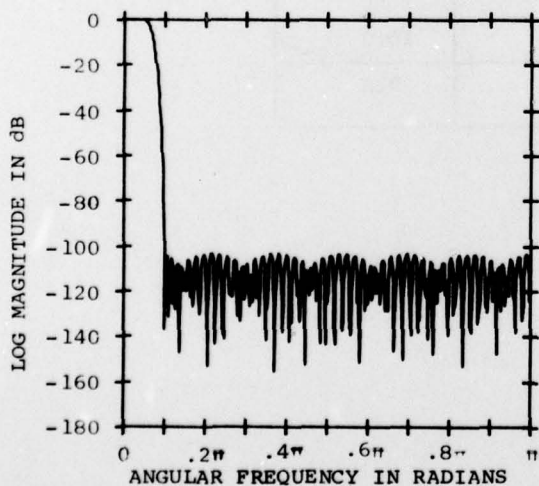


Fig. 2. FREQUENCY RESPONSE FOR THE ONE-STAGE IMPLEMENTATION OF EXAMPLE 1

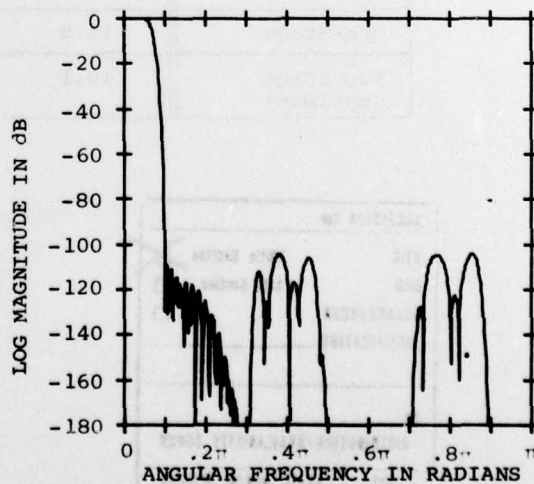


Fig. 3. FREQUENCY RESPONSE FOR THE TWO-STAGE IMPLEMENTATION OF EXAMPLE 1

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2. GOVT ACCESSION NO.	3. REPORT NUMBER	12/6p.1	
4. TITLE (and Subtitle)		5. TYPE OF REPORT & PERIOD COVERED	
ALIASING ERROR IN THE MULTIRATE IMPLEMENTATION OF NARROWBAND FILTERS.		9/ Interim rept.	
6. AUTHOR(s)		6. PERFORMING ORG. REPORT NUMBER	
Fred/Mintzer Bede/Liu		15/ AFOSR-76-3083	
9. PERFORMING ORGANIZATION NAME AND ADDRESS		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS	
Princeton University Department of Electrical Engineering Princeton, NJ 08540		611028 2304/A6	
11. CONTROLLING OFFICE NAME AND ADDRESS		12. REPORT DATE	
Air Force Office of Scientific Research/M Bolling AFB, DC 20332		11/ May 1977	
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		13. NUMBER OF PAGES	
		4	
15. SECURITY CLASS. (of this report)		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE	
UNCLASSIFIED			
16. DISTRIBUTION STATEMENT (of this Report)			
Approved for public release; distribution unlimited			
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)			
18. SUPPLEMENTARY NOTES			
1977 IEEE INTERNATIONAL CONFERENCE ON ACOUSTICS, SPEECH, AND SIGNAL PROCESSING, pp 105-108, May 1977			
19. KEY WORDS (Continue on reverse side if necessary and identify by block number)			
digital filtering multirate filtering decimation narrowband filtering interpolation aliasing error			
20. ABSTRACT (Continue on reverse side if necessary and identify by block number)			
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20. Abstract

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